Resistive wall wakefield and beam interactions for the femtosecond source

The transverse wakefield from the resistive wall for a circular pipe of radius b, length L, electrical conducticvity $\sigma_{conductivity}$ is given by [Handbook of Accelerator Physics and Engineering]:

$$W_1(z) = \frac{c}{\pi b^3}\sqrt{\frac{Z_0}{\pi\,\sigma_{conductivity}}}\,\frac{L}{\sqrt{z}}$$

For a charge distribution ρ(z') the wake is then:

$$W_1(z) = \frac{c\,L}{\pi b^3}\sqrt{\frac{Z_0}{\pi\,\sigma_{conductivity}}}\int_z^{\infty}\frac{\rho(z')}{\sqrt{|z-z'|}}\,dz'$$

And for a Gaussian bunch:

$$W_1(\text{bunch}, z) = \frac{c\,L}{\pi b^3}\sqrt{\frac{Z_0}{\pi\,\sigma_{conductivity}}}\,\frac{1}{\sigma_{bunch}\sqrt{2\pi}}\int_z^{\infty}\frac{e^{-\frac{z'^2}{2\sigma_{bunch}^2}}}{\sqrt{|z-z'|}}\,dz'$$

The deflecting voltage for a bunch of charge Ne, and offset y_0 is:

$$V_{transverse}(\text{bunch}, z) = N\,e\,y_0\,W_1(\text{bunch}, z)$$

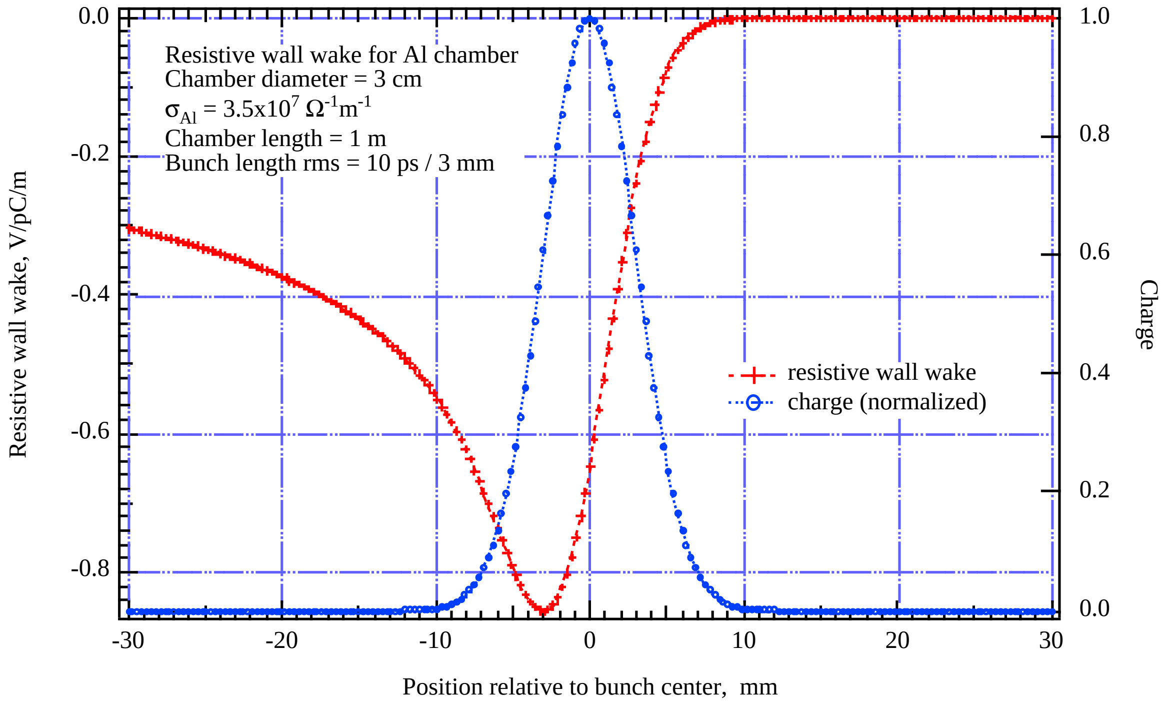
And the angular deflection of a beam of energy E (eV)is:

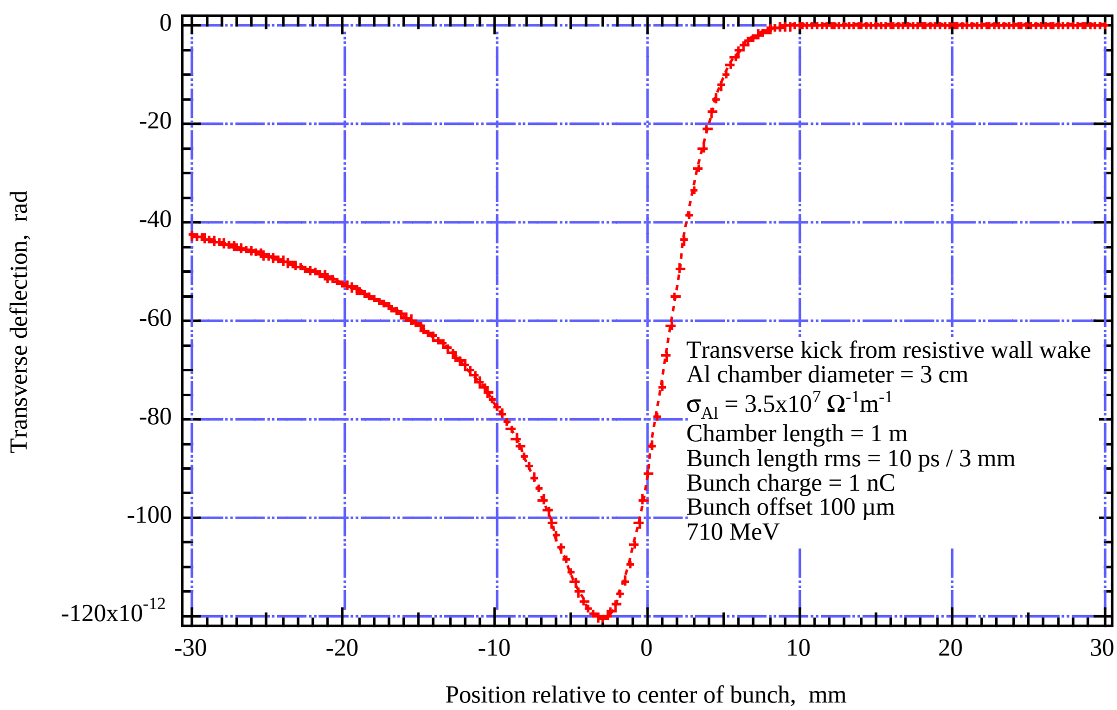
$$\Delta\,y'_1(\text{bunch}, z) = \frac{N\,e\,y_0}{E}\,\frac{c\,L}{\pi b^3}\sqrt{\frac{Z_0}{\pi\,\sigma_{conductivity}}}\,\frac{1}{\sigma_{bunch}\sqrt{2\pi}}\int_z^{\infty}\frac{e^{-\frac{z'^2}{2\sigma_{bunch}^2}}}{\sqrt{|z-z'|}}\,dz'$$

For the parameters of Table 1, the wakefield is shown in Figure 1 together with the charge distribution. The deflection angle for the same parameters is shown in figure 2.

Table 1. Initial parameter set - first arc

Bunch length, σ	10 ps
Bunch length, σ	3 mm
Chamber radius	1.5 cm
Chamber length	1 m
Chamber conductivity	$3.5\text{x}10^7$ (Al)
Bunch offset	100 μm
Number of electrons	$6.24\text{x}10^9$
Beam energy	710 MeV



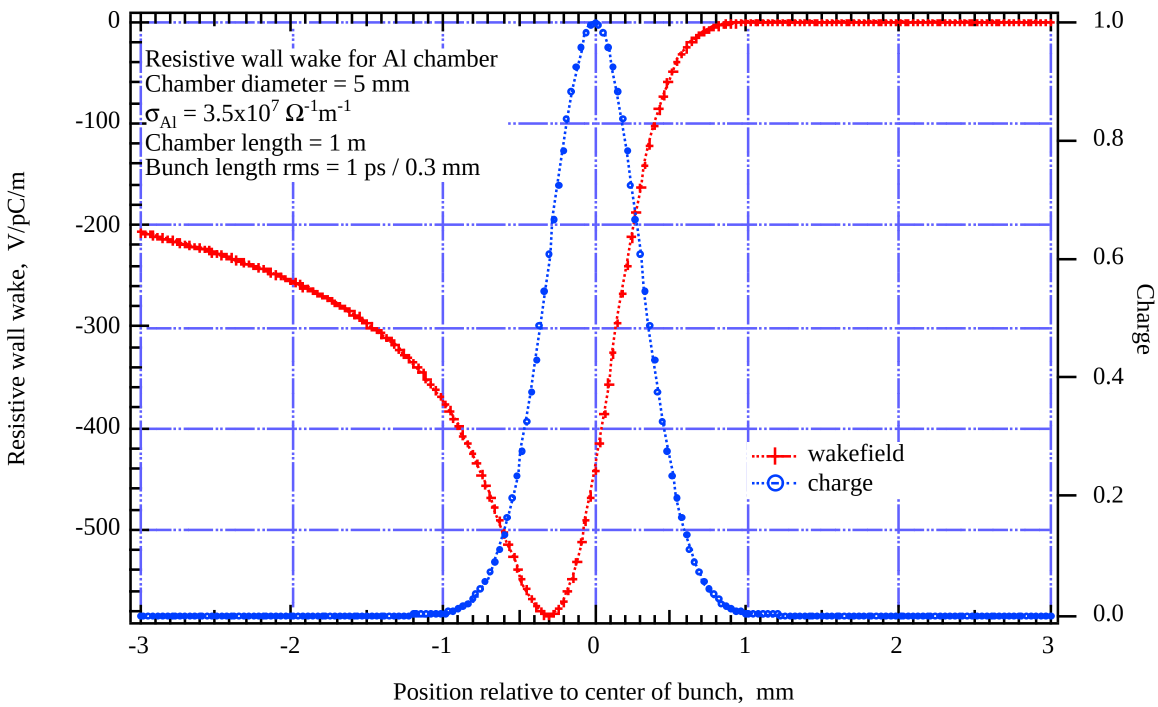


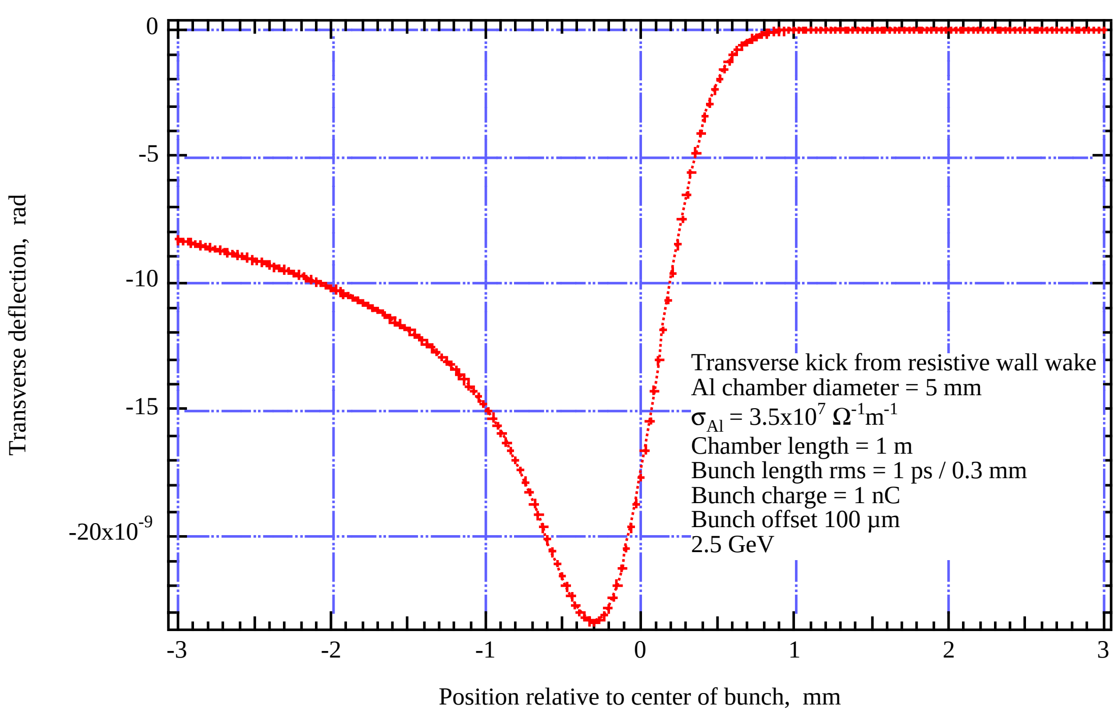
The deflection is small, and only 12 nm distortion in σ_y over 100 m of 3 cm diameter beampipe

We now explore the parameter regime, that in the final arc:

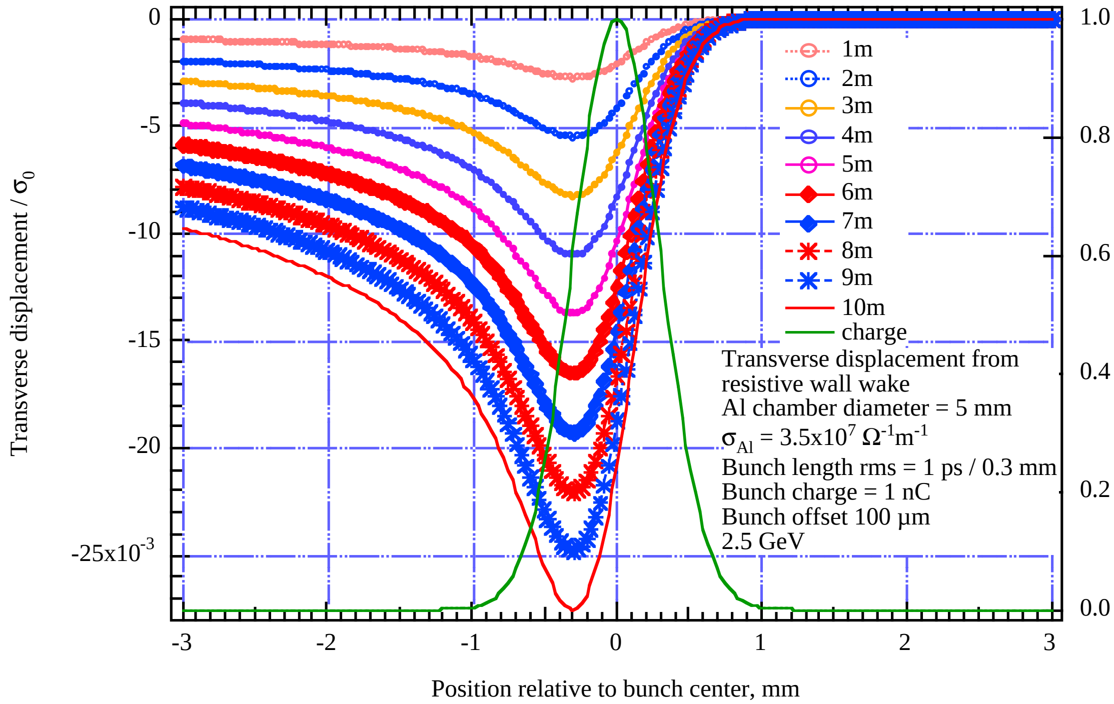
Table 2. Final arc (photon production) parameter set

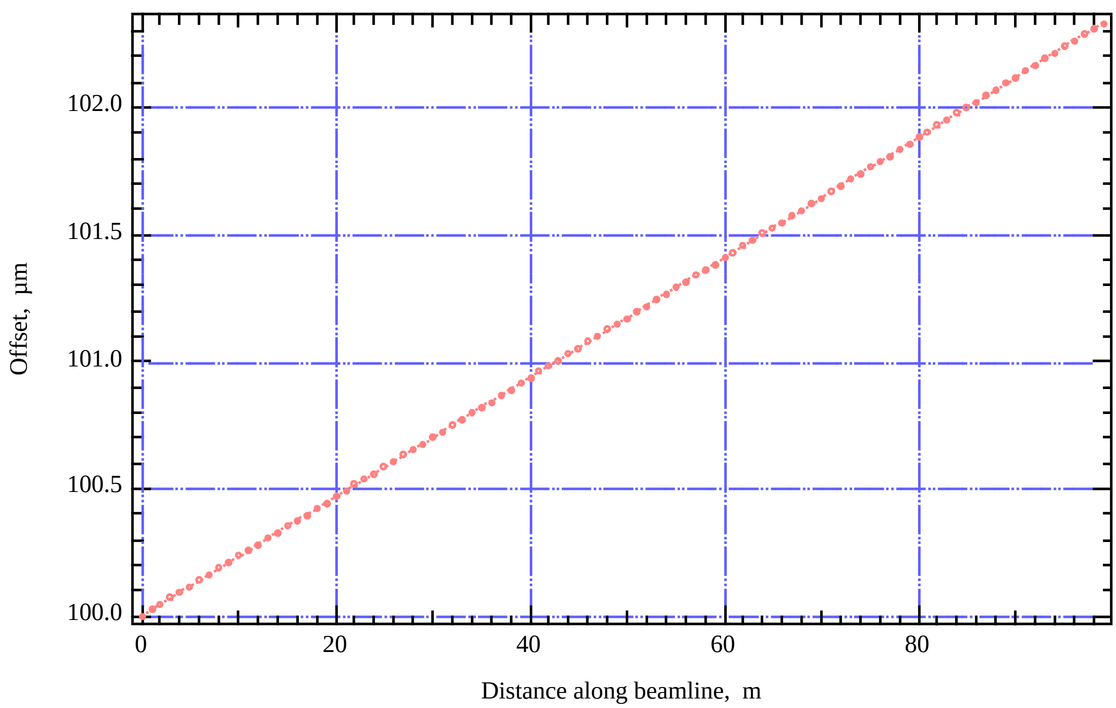
Bunch length, σ	1 ps
Bunch length, σ	0.3 mm
Chamber radius	2.5 mm
Chamber length	1 m
Chamber conductivity	3.5×10^7 (Al)
Bunch offset	100 µm
Number of electrons	6.24×10^9
Beam energy	2.5 GeV
Beamsize σ_y	8.5 µm


Resistive wall wake for Al chamber
Chamber diameter = 5 mm
$\sigma_{Al} = 3.5x10^7\ \Omega^{-1}m^{-1}$
Chamber length = 1 m
Bunch length rms = 1 ps / 0.3 mm
wakefield
charge
Resistive wall wake, V/pC/m
Charge
Position relative to center of bunch, mm
0
-100
-200
-300
-400
-500
1.0
0.8
0.6
0.4
0.2
0.0
-3
-2
-1
0
1
2
3


Transverse kick from resistive wall wake
Al chamber diameter = 5 mm
$\sigma_{Al} = 3.5x10^7\ \Omega^{-1}m^{-1}$
Chamber length = 1 m
Bunch length rms = 1 ps / 0.3 mm
Bunch charge = 1 nC
Bunch offset 100 µm
2.5 GeV
Transverse deflection, rad
Position relative to center of bunch, mm
0
-5
-10
-15
-20x10^{-9}
-3
-2
-1
0
1
2
3

The position of the bunch along the photon production straight is:





The distortion is approximately 0.28% for 100 μm offset in 1m of chamber height 5 mm.

0.14 percent for a 50 μm bunch offset in 1 m of chamber height 5 mm.

This suggests that the maximum bunch offset per meter of narrow-bore id (5 mm height) is ~ 50 μm to maintain bunch distortions at the one percent level over a total of 10 m of narrow-bore chamber.

The vacuum chamber height should be increased between id's, taking care to avoid strong wakefields from tapers.

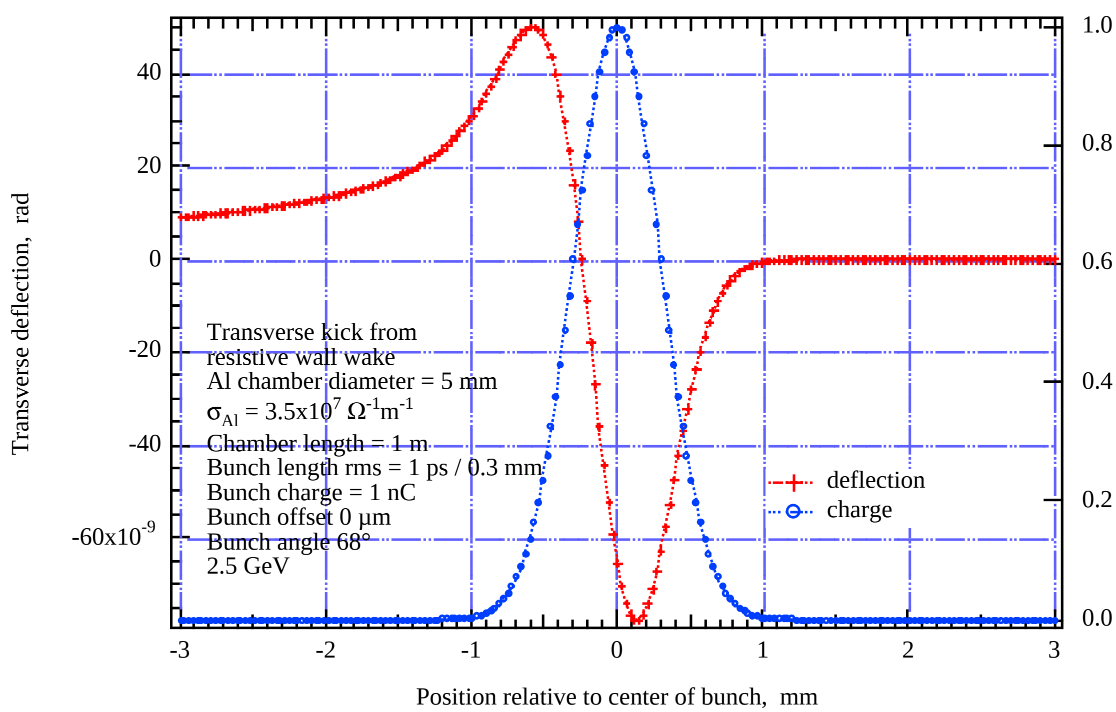
Alternatively, the vacuum chamber height may be increased to 6.3 mm to allow a bunch offset of 100 μm with ~ one percent distortions along the bunch in 10 m of narrow-bore chamber.

Now introduce the angular position variation within the bunch, as in the photon production section dipole magnets.

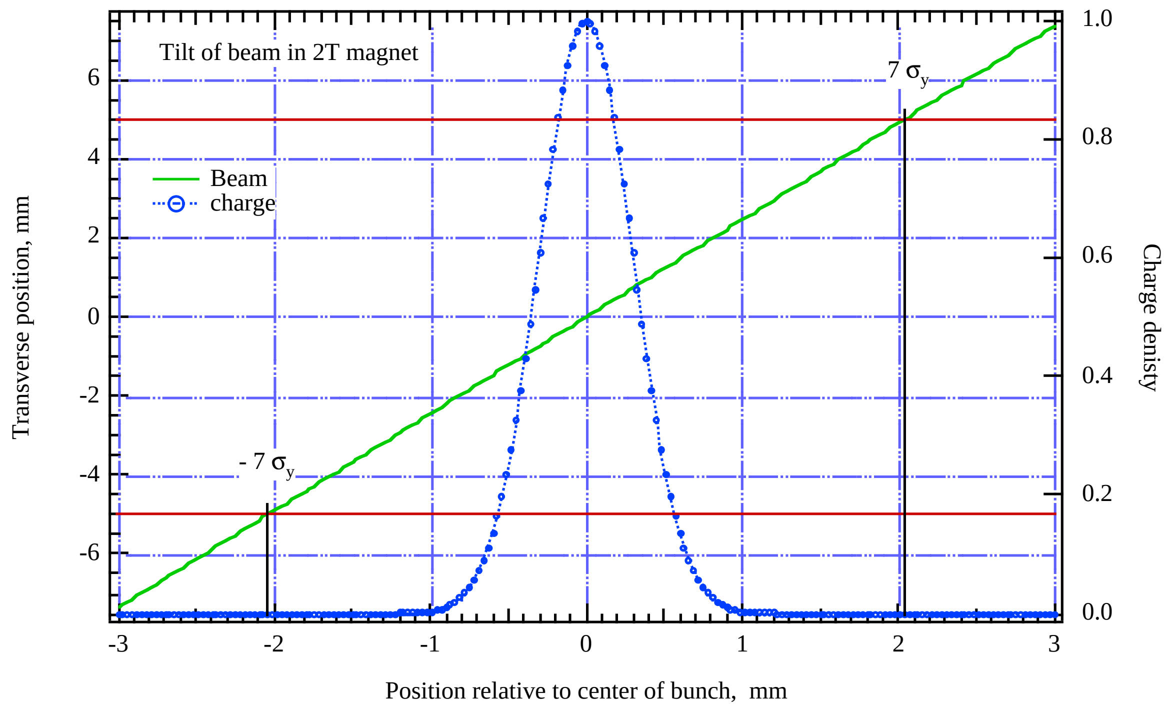
The effects arising from the tilt of the bunch are significant. The bunch angle in the bend magnets is 68° (1.19 rad) and rms bunch length 0.3 mm (1 ps), so the beam at $1\sigma_z$ is 278 μm displaced from the axis. The apparent (projected on machine axis) σ_z is reduced to 0.378 of that in the bunch axis.

$$\delta y\,(z) = \frac{e\,U}{E}\,\sqrt{\beta_{cavity}\,\beta_{bend}}\,\sin\left(k_{rf}\,z\right)$$

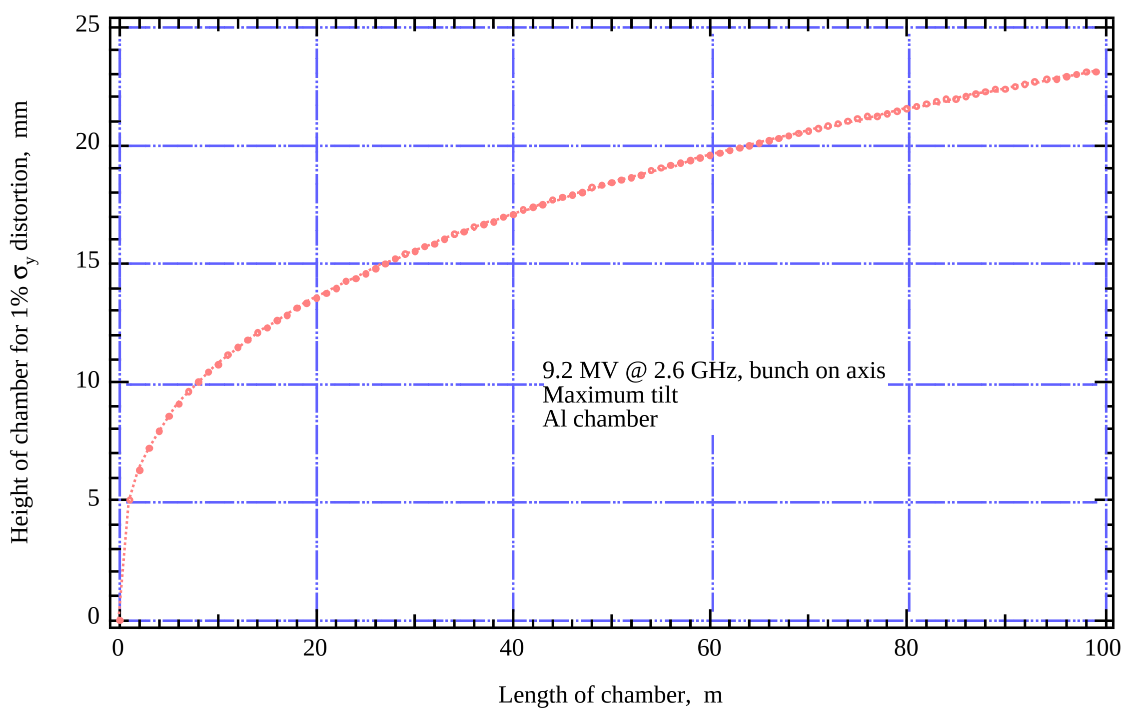
$$\frac{dy}{dz}(0) = k_{rf}\,\frac{e\,U}{E_{beam}}\,\sqrt{\beta_{cavity}\,\beta_{bend}} = \frac{2\,\pi\,2.6\mathrm{x}10^9}{8}\,\frac{9.2\mathrm{x}10^6}{9}\,\sqrt{50\mathrm{x}3} = 2.46$$



The bunch distortion is 1 % of σ_y in 1 m of 5 mm height dipole vacuum chamber.

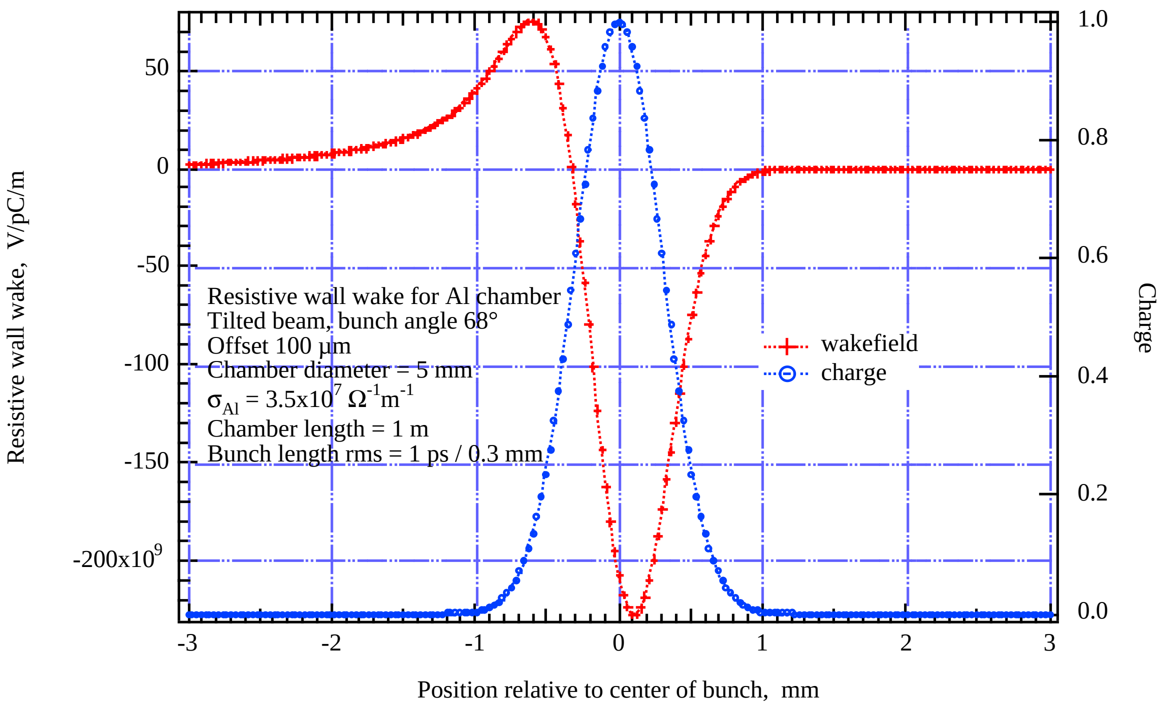


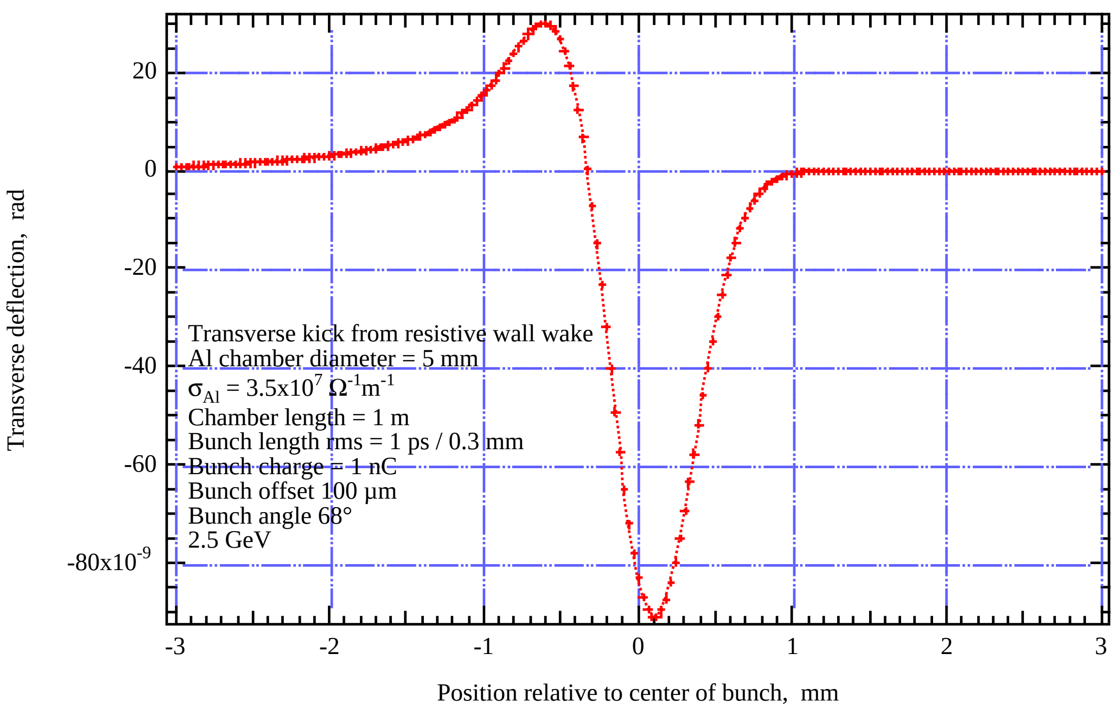
Tilt of beam in 2T magnet
Beam
charge
7 σ_y
- 7 σ_y
Transverse position, mm
Charge denisty
Position relative to center of bunch, mm



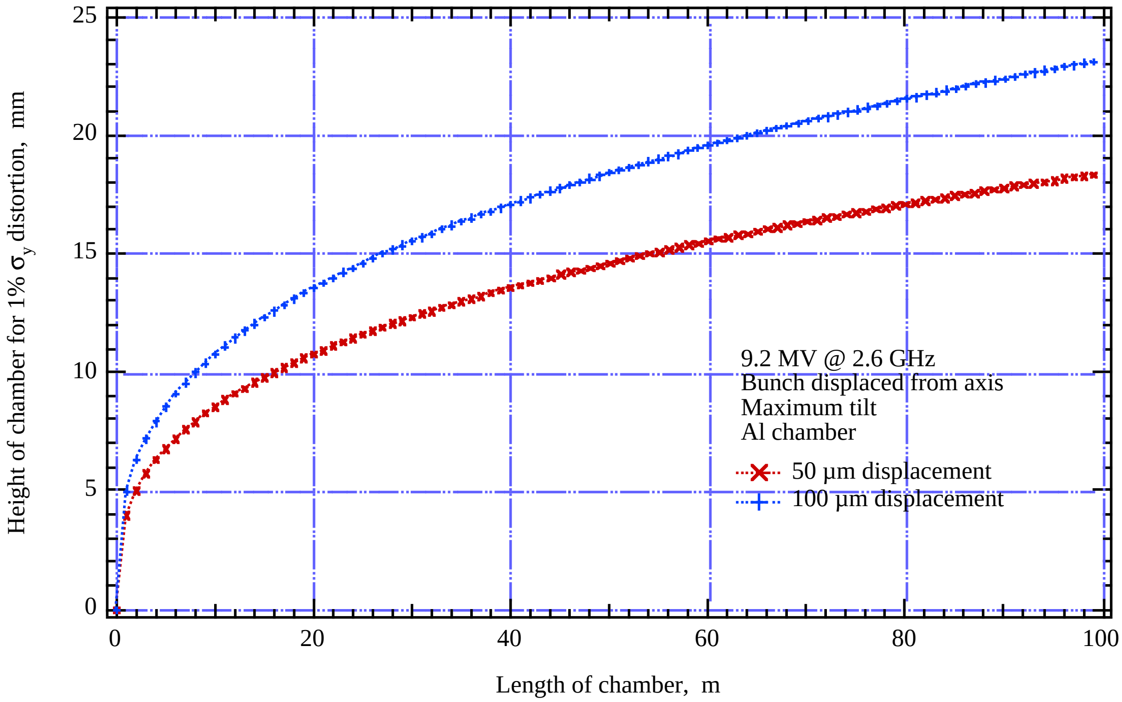
9.2 MV @ 2.6 GHz, bunch on axis
Maximum tilt
Al chamber
Height of chamber for 1% σ_y distortion, mm
Length of chamber, m

Now include a 100 µm offset in the bunch:





The bunch distortion is 1 % of σ_y in 1 m of 5 mm height dipole vacuum chamber, with 100 µm offset.



This analysis does not include effects from:

- Flat beampipe (20% effect)
- Focussing and ß-tron motion
- CSR or other bunch distortions
- Longitudinal effects (energy spread)
- Other energy passes
- Other bunch lengths